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STUDIES OF THE DYNAMICS OF TRACKED VEHICLES

by

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13. ABSTRACT

A field test program was conducted with four tracked vehicles to determine how strongly the presence of the track affects ride dynamics and to guide in the development of a mathematical model. The vehicles were towed over an assortment of obstacles, first with tracks installed and then with tracks removed. A direct comparison of dynamic responses under these two conditions indicated that the influence of the track is strongly dependent on velocity, and that mathematical models of tracked vehicles must incorporate a track contribution. A mathematical model that portrays essential features of track mechanics without excessive detail was developed.

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FOREWORD

The study reported herein was conducted in 1970-71 by personnel of the Mobility Research Branch, Mobility and Environmental Division, U. S. Army Engineer Waterways Experiment Station (WES), in furtherance of DA Project 1T061102B52A, "Research in Military Aspects of Terrestrial Sciences," Task 01, "Military Aspects of Off-Road Mobility," under the sponsorship and guidance of the Research, Development and Engineering Directorate, U. S. Army Materiel Command.

The tests were conducted under the general supervision of Messrs. W. G. Shockley, S. J. Knight, and A. J. Green and under the direct supervision of Dr. A. S. Lessem and Mr. N. R. Murphy, Jr., who developed the mathematical model and prepared this report.

COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE, were Directors of the WES during this study and preparation of this report. Mr. F. R. Brown was Technical Director.

CONTENTS

CONVERSION FACTORS, BRITISH TO METRIC UNITS OF MEASUREMENT

British units of measurement used in this report can be converted to metric units as follows:

Multiply	By	To Obtain
inches	2.54	centimeters
feet	0.3048	meters
square inches	6.4516	square centimeters
inches per second	2.54	centimeters per second
feet per second	0.3048	meters per second
miles (U. S. statute) per hour	1.609344	kilometers per hour
pounds (force)	4.4482	newtons
pounds (mass)	0.4535924	kilograms
kips (force)	4.4482	kilonewtons
pounds per inch	175.1	newtons per meter
pounds per square inch	6.8948	kilonewtons per square meter

SUMMARY

A field test program was conducted with four tracked vehicles to determine how strongly the presence of the track affects ride dynamics and to guide in the development of a mathematical model. The vehicles were towed over an assortment of obstacles, first with tracks installed and then with tracks removed. A direct comparison of dynamic responses under these two conditions indicated that the influence of the track is strongly dependent on velocity, and that mathematical models of tracked vehicles must incorporate a track contribution.

A mathematical model that portrays essential features of track mechanics without excessive detail was developed.

STUDIES OF THE DYNAMICS OF TRACKED VEHICLES

PART I: INTRODUCTION

Background

1. In recent years, computer simulation of vehicle dynamics has become an important part of the vehicle design process. The development of mathematical models depicting the dynamics of wheeled vehicles has received considerable attention;[1] but compared with that effort, much less study has been directed to models for tracked vehicles. The essential difference between models for these two classes of vehicles is the representation of the traction elements. Several useful models of pneumatic tires have been developed;[2] however, no models of tracks have yet appeared, a fact easily understood because of the great complexity of track physics.

2. The most frequent approach[3,4] has been to neglect the presence of the tracks and to imagine that the vehicle suspension, i.e. the road wheels, "sees" the terrain profile directly. This approach is expedient and reasonable for those situations, such as relatively smooth terrains, in which dynamics do not limit mobility. Another approach[5] incorporates a terrain modification process intended to simulate the smoothing action of the track. The modified terrain is then treated as the input to the suspension system. Both approaches fail to give realistic results under circumstances of intense dynamics.

3. An important effort has been made[6] to develop a more realistic track model by depicting the role of track tension. The outcome was a computation procedure that can account for suspension deflections due to track tension, in addition to those due to ground contact. Computations have not been performed because of the excessive time required for iterative solutions of the coupled nonlinear equations. This fact illustrates the principal barrier confronting the development of a realistic track model: if at all possible, the track model should not be much

more complex than the model depicting the remainder of the vehicle.

Purpose

4. The study reported herein was intended to determine directly from tests with a group of tracked vehicles (a) whether an effort to develop a track model was justified, and (b) if there are reasonably broad regions in the ranges of vehicle velocities and obstacle dimensions for which neglect of track contributions to hull dynamics would be acceptable. If such regions could be adequately delineated and were not sufficiently broad, development of a model for tracklayer dynamics that would be as simple as possible would be attempted by appealing to laboratory tests with actual vehicles for the determination of parameters. These tests would involve measuring deflections of track and suspension elements with the vehicles stationary.

Scope

5. Field tests were conducted with four tracked vehicles of significantly different weights and track-suspension properties. Although a study of these vehicles with a broad class of obstacle types was planned, severe testing problems limited the program to a single half-round obstacle 8 in.* in diameter, and to 8-in.-deep ditches that were 12, 24, 36, and 48 in. wide. Approximately 360 tests were conducted in which the vehicles were towed across these obstacles, first with tracks installed and later with tracks removed. A wide range of traversal speeds was desired, but operational problems limited the maximum speed to approximately 8 fps.

6. A mathematical model for tracked vehicle dynamics was formulated that bridged the gap between one that completely neglected the track and one based on all contributing aspects of track physics.

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* A table of factors for converting British units of measurement to metric units is given on page vii.

PART II: EXPLORATORY FIELD STUDY OF TRACKED VEHICLE DYNAMICS

7. The desired outcome of the field program was to determine, directly from tests with tracked vehicles, the necessity for inclusion of track effects in models of tracked vehicle ride dynamics. It can be seen in advance, of course, that tracks do contribute to dynamics, in the very least by modifying the terrain profile "seen" by the suspension elements, but the magnitude of their contribution has only been guessed. Because physically motivated models depicting track effects will likely be greater in complexity than the remainder of the vehicle model, it was desirable to determine whether or not track models are really required for realistic analysis of ride dynamics, not so much to determine a "yes" or "no" answer, but rather to see if a broad range of operational circumstances could be determined for which track effects could be neglected.

Vehicles Used

8. From the complement of tracked test vehicles available at the U. S. Army Engineer Waterways Experiment Station (WES), four were selected on the basis of variation in weight. The vehicles and their unloaded weights, track features, and suspension types are tabulated below.

Vehicle	Unloaded Weight lb	Track Features	Suspension Type
M29	4,771	Rubberized; 20-in. width; 8 bogies per side in pairs	Transverse leaf per pair of bogies
M114	12,537	Rubberized; 16-in. width; 4 road wheels per side	Torsion bar, independent
M113	19,865	Single-pin track; 15-in. width; 5 road wheels per side	Torsion bar, independent
M4	31,400	Double-pin track; 4 bogies and 1 load-bearing idler per side	Side springs; axle attached directly to hull

There was essentially no other choice of vehicle characteristics readily

available, resulting in an uncontrolled spread of track and suspension characteristics.

Test Setup and Procedures

9. The basic idea for all testing was to tow each vehicle across a pertinent array of obstacles, first with tracks installed and later with tracks removed. By comparing time histories of accelerations and other motions, the importance of track effects could be judged directly. When the vehicles were untracked, an additional weight equal to half the weight of the removed tracks was added to the hull weight.

10. Each vehicle was instrumented to record the following dynamic variables:

- a. Vertical and horizontal accelerations at the center of gravity.
- b. Pitch and roll at the center of gravity.
- c. Drawbar pull on the towline.
- d. Horizontal velocity and distance traveled.

11. Vehicle traversal of an assortment of obstacles capable of inducing bounce, roll, and pitch motions of increasing severity was planned, but could not be done because of severe mechanical problems. For example, towing the vehicles very far before they wandered off the desired path proved impossible. The maximum obstacle height was limited not by severity of dynamics, but by the requirement to avoid striking the sprocket of the untracked vehicles.

12. The net outcome was the reduction of the obstacle field to a single, 8-in.-diam, half-round obstacle transverse to the direction of vehicle travel, and a sequence of ditches 8 in. deep and 12, 24, 36, and 48 in. wide. Both obstacle types were impacted simultaneously by right and left sides of the vehicles in order to suppress roll motions, a requirement for straight-line traversal of the obstacles.

13. Each vehicle was towed over each obstacle at nominal speeds of 2, 5, and 8 fps. Control of vehicle speed is always a problem in field studies of ride dynamics, and was especially so in this program.

As many tests were conducted as were required to obtain the desired speed within ±10 percent. A total of 360 tests were conducted in the program. Three of the vehicles in a variety of situations during the test program are shown in fig. 1.

Data Processing

14. In earlier work at the WES,[7] a useful descriptor of short-term obstacle-induced dynamics was developed. This descriptor was used to analyze data taken during the tracked vehicle program. The original development of the descriptor is as follows:

a. A distance factor D is computed as

$$D = 1.6 \times (\text{vehicle wheelbase})$$

b. With a time history of vehicle horizontal velocity $v(t)$ obtained during an obstacle traversal test, a time factor T is computed from

$$D = \int_0^T v(t)\, dt$$

c. With a time history of acceleration $a(t)$ at a point of interest, a vibration descriptor A is computed as

$$A = \sqrt{\frac{1}{T}\int_0^T [a(t)]^2\, dt}$$

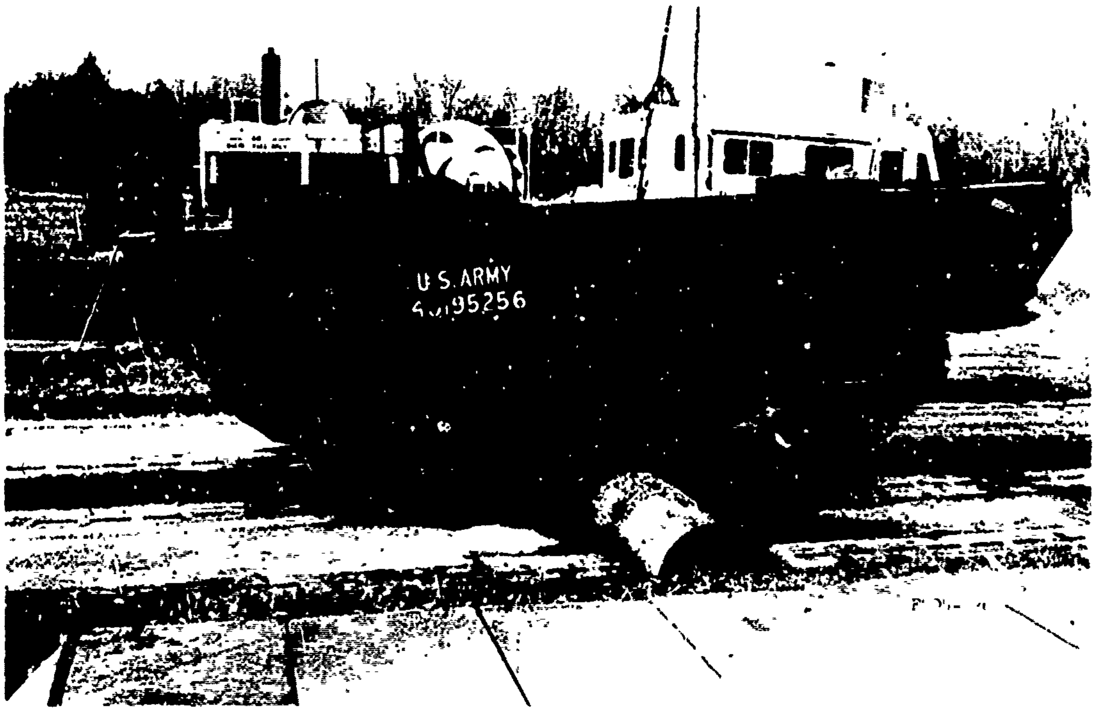
This quantity is the root mean square (RMS) acceleration.

d. The average velocity V is computed as

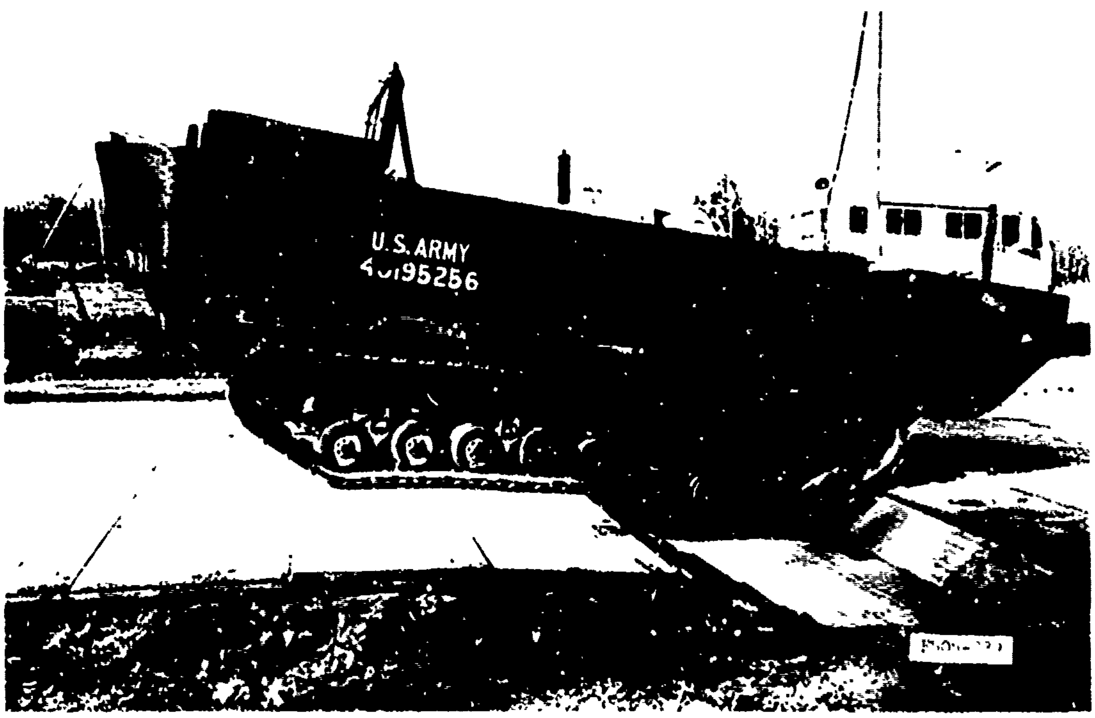
$$V = \frac{D}{T}$$

e. For a range of traversal velocities and corresponding accelerations, A is plotted versus V for each obstacle.

15. Plots of this kind are useful for standardizing the description

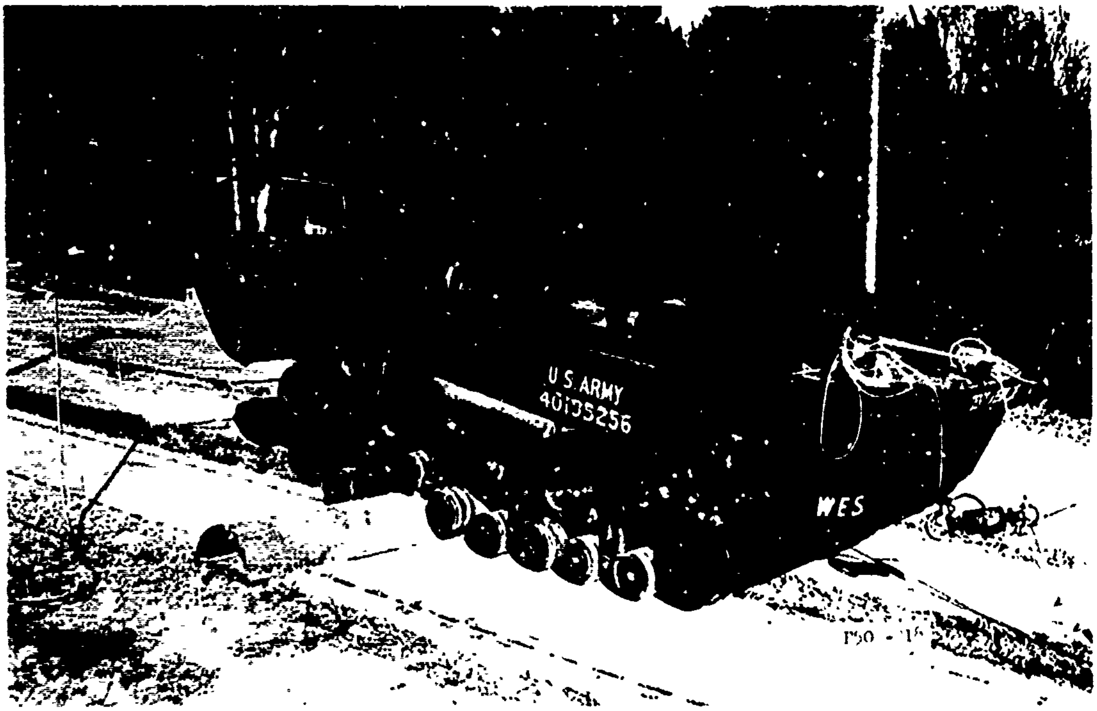


a. Tracked M29 traversing 8-in. half-round obstacle

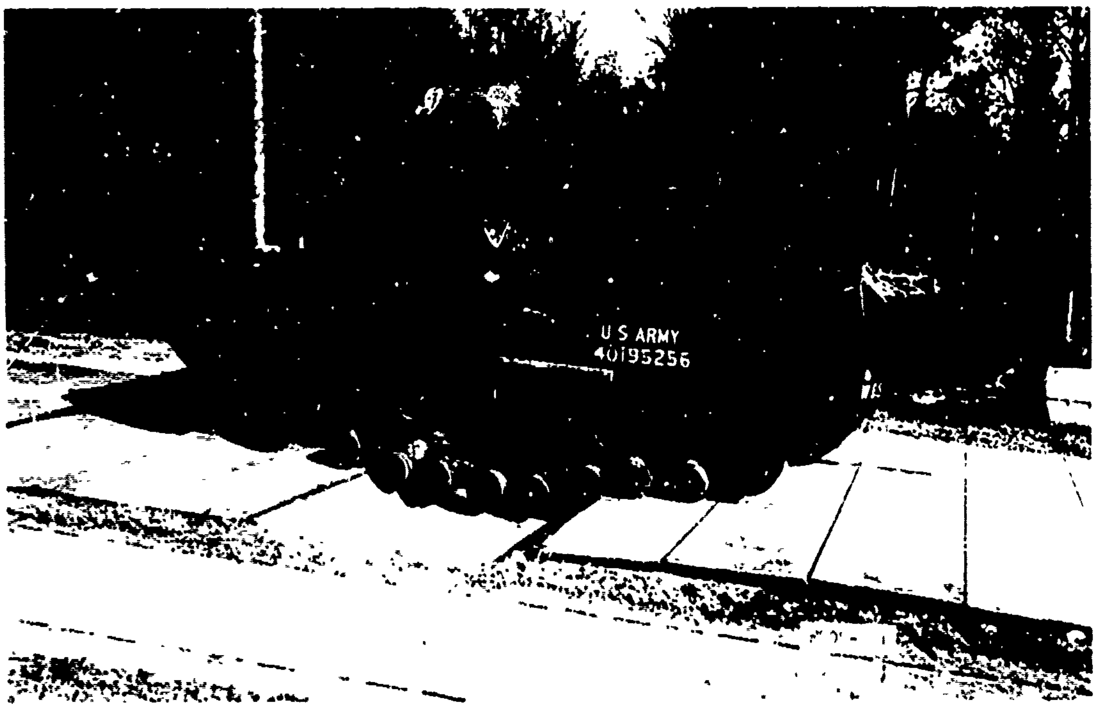


b. Tracked M29 traversing 8- by 48-in. ditch

Fig. 1. M29, M113, and M4 test vehicles traversing test obstacles (sheet 1 of 5)

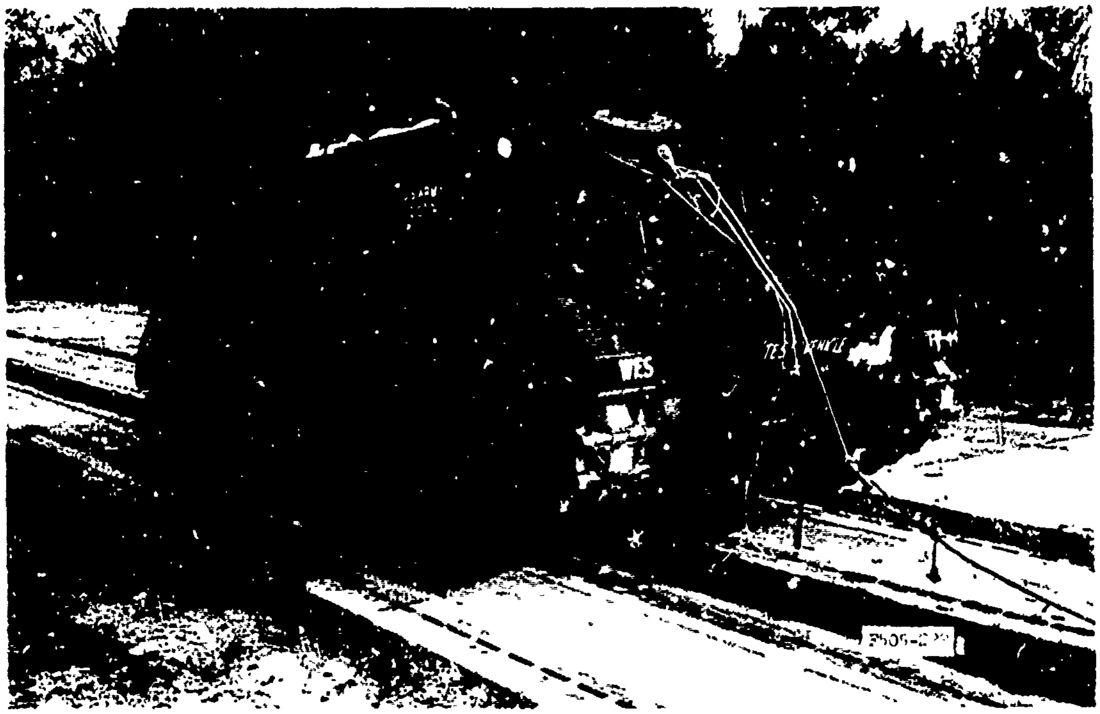


c. Untracked M29 traversing 8-in. half-round obstacle

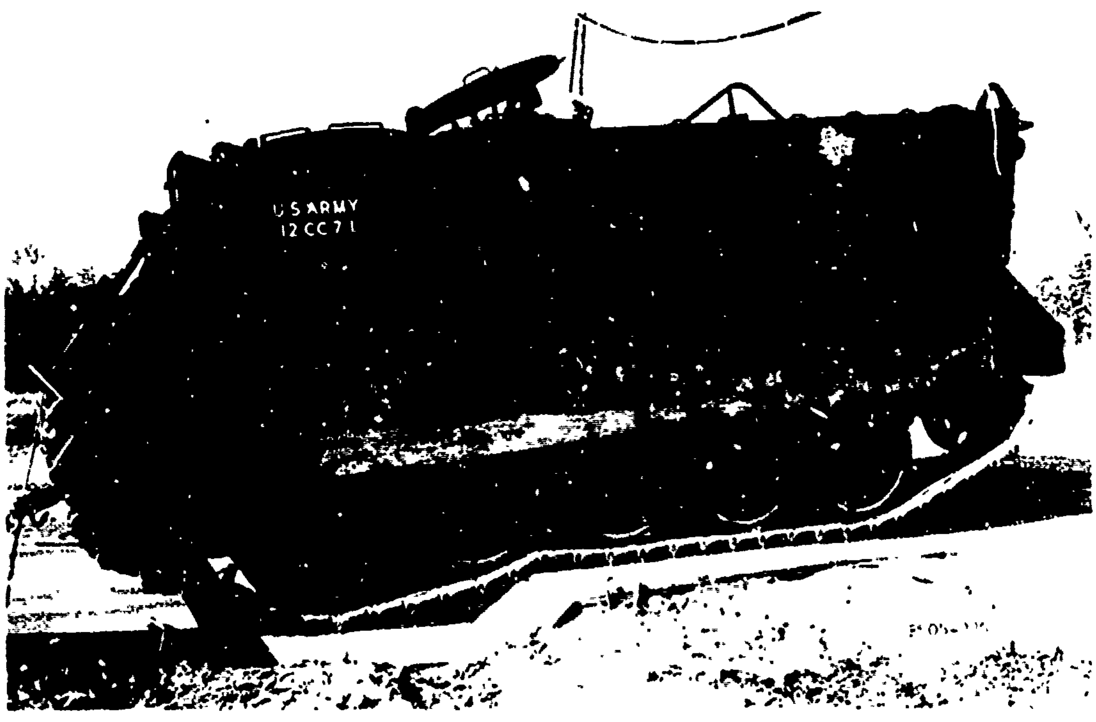


d. Untracked M29 traversing 8- by 48-in. ditch

Fig. 1 (sheet 2 of 5)

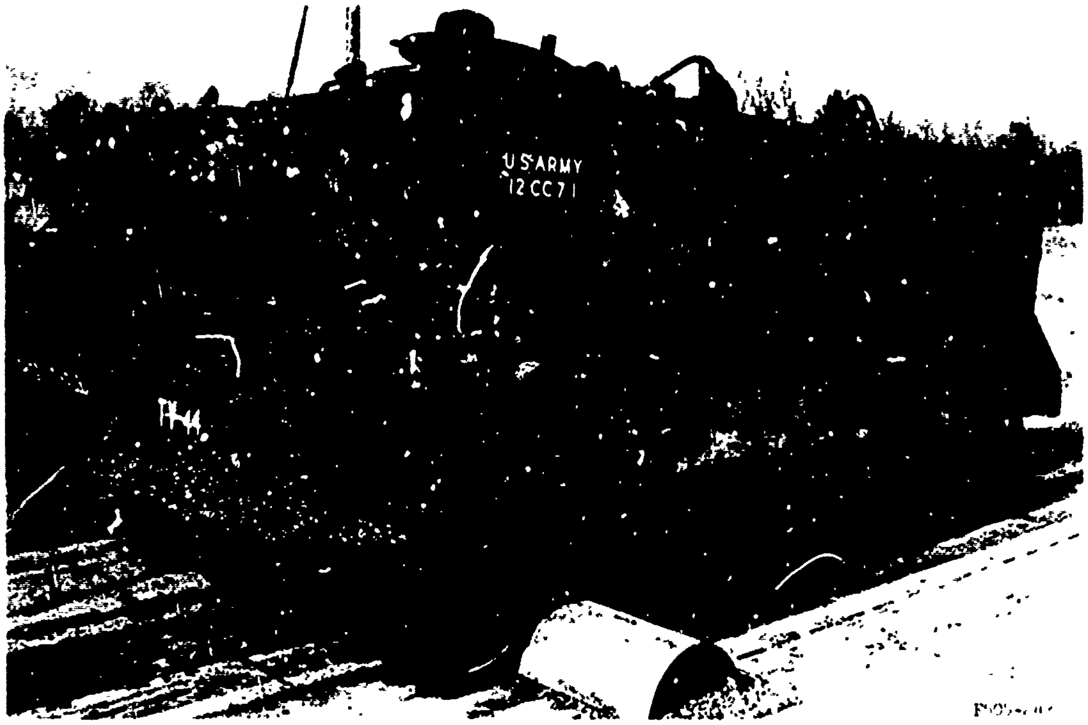


e. Tracked M113 traversing 8-in. half-round obstacle

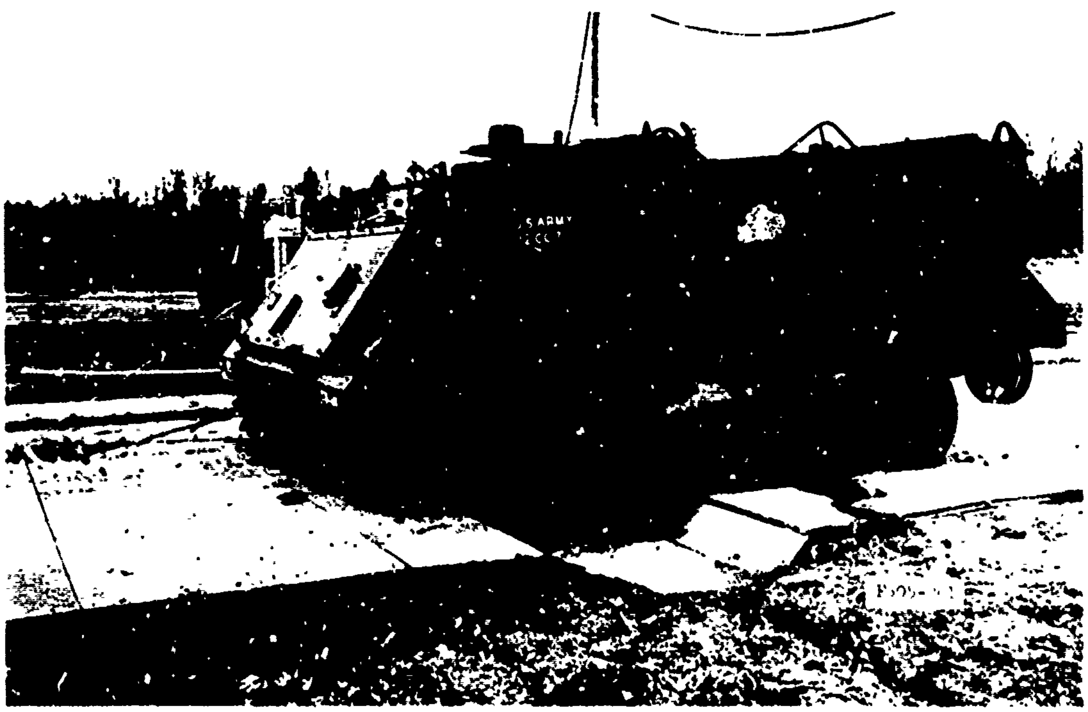


f. Tracked M113 traversing 8- by 48-in. ditch

Fig. 1 (sheet 3 of 5)

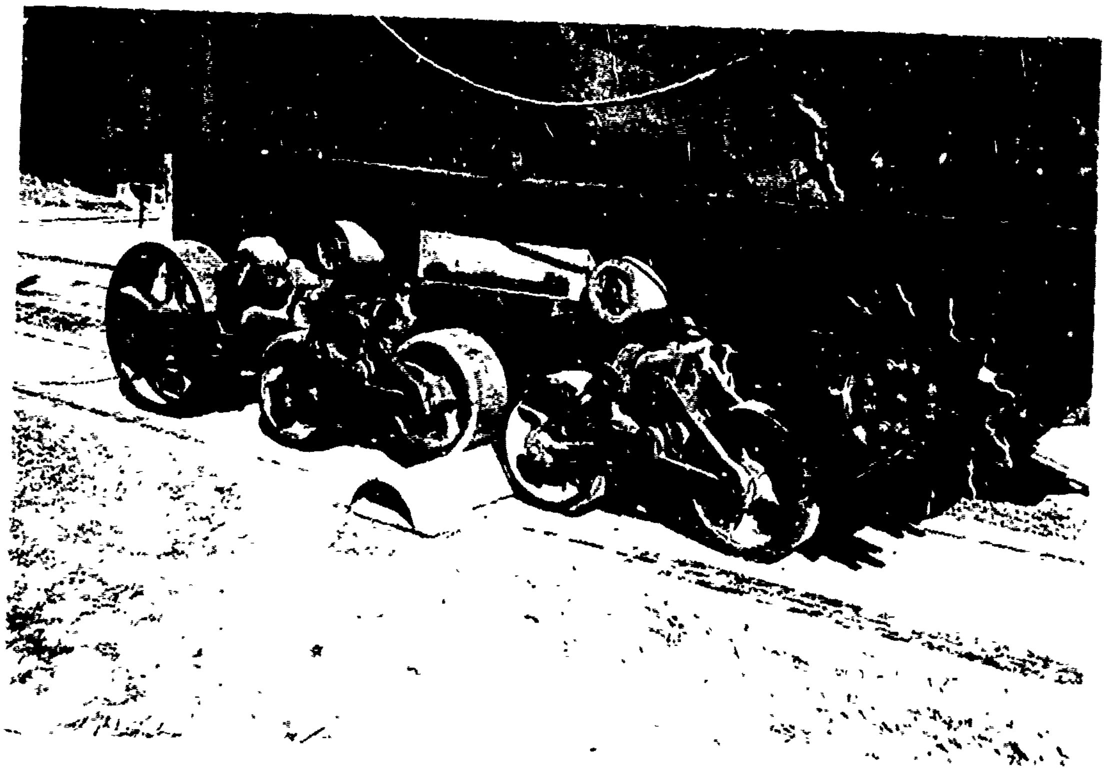


g. Untracked M113 traversing 8-in. half-round obstacle



h. Untracked M113 traversing 8- by 48-in. ditch

Fig. 1 (sheet 4 of 5)



i. Untracked M4 traversing 8-in. half-round obstacle

Fig. 1 (sheet 5 of 5)

of short-term dynamics in the presence of inevitable velocity fluctuations. They are also useful for comparing the performances of different vehicles in similar operational circumstances, or, as in the case of the tracked vehicle program, for assessing the influence of structural changes on the dynamics of a given vehicle.

16. In adapting the basic idea for this descriptor to the analysis of the tracked vehicle data, the following three considerations were of importance:

a. Because of the variety of track and road-wheel configurations involved, definition of an effective "wheelbase" for computation of the distance factor D was difficult. Because a vehicle's performance with tracks was to be compared with its performance without tracks, and results for one vehicle were not to be compared with those for another vehicle, a value for D was selected without regard to the wheelbase of any vehicle, but was characteristic of the distance traversed while the vehicle was undergoing significant dynamics. The value selected for D was 15 ft.

b. In characterizing short-duration vibrational activity, the point of interest was the center of gravity of the hull of each vehicle. In anticipation of significant contributions from longitudinal accelerations, a "composite" descriptor A_C was computed from the vertical and horizontal acceleration-time histories $a_V(t)$ and $a_H(t)$, respectively, as

$$A_c = \sqrt{\frac{1}{T}\int_0^T \left[a_V^2(t) + a_H^2(t)\right] dt}$$

In addition, a "vertical" descriptor A_V was computed as

$$A_V = \sqrt{\frac{1}{T}\int_0^T a_V^2(t)\ dt}$$

c. The dynamics descriptor was originally developed for wheeled vehicles and displayed an essentially linear dependence on velocity for a given obstacle height. No such linear relation appeared in the tracked vehicle data, and no attempt was made to smooth the curves of A_C or A_V versus V ; data points were joined by straight lines to show trends.

17. Dynamics data recorded on magnetic tape during testing were played back to an analog computer for processing. Because interest was centered on hull dynamics, where frequencies in the range of 0.5 to 3 Hz are important, the data were smoothed with a low-pass filter with a cut-off frequency of 10 Hz. This served to remove spurious contributions of engine and flexural vibrations to the descriptors.

Test Results

18. Plots of A_c versus V and A_v versus V were prepared for each vehicle traversing each obstacle in both tracked and untracked configurations (plates 1-4). Because the performance of each normally tracked vehicle was to be compared with its performance with tracks removed, the ratios of A_c with tracks on to A_c with tracks off and of A_v with and without tracks were plotted against V (plates 5-8). Putting the data in this form is useful because the condition of most interest--the equality of tracked and untracked vehicle responses--appears as a ratio of unity. Values of this ratio above unity indicate tracked vehicle responses more severe than untracked, and values below unity indicate tracked vehicle responses smoother than untracked.

19. Although the mechanical difficulties encountered during the field program significantly controlled the scope of the test program, some basic trends can be seen. It is apparent that the suspension system plays an important part in determining the extent to which the track affects dynamics. The M113 and M114, having suspensions composed of torsion bars and trailing arms with large road wheels, exhibited generally smoother responses than those of the other vehicles, with less difference between tracked and untracked conditions. The M4, with its axles attached directly to the hull, was most sensitive to the absence of tracks. The M29, with its very compliant transverse leaf springs, displayed a resonance effect in which the tracked response was much more severe than the untracked response at certain velocities.

20. Although no clear trend emerged, in most cases the influence of the track in smoothing the dynamic responses was more pronounced at

low velocities in the range studied. This was especially apparent where vertical obstacles and ditches were traversed whose width was on the order of the road-wheel spacing. Although the highest velocity studied in this program (8 fps) was rather low, even for off-road conditions, it was surprising to note that the smoothing effect of the track quickly diminished with increasing velocity. For the cases of ditches longer than the road-wheel spacing, no such trend was apparent.

21. The net outcome of studying plates 5-8 is the conclusion that the occurrence of intervals where tracked and untracked responses are the same is rare, and that an effort must be made to formulate a useful track model. Departures from equality, or near equality, of responses are significant, and they indicate the inadequacy of models that ignore the track effect.

PART III: A MATHEMATICAL MODEL FOR TRACKED VEHICLE DYNAMICS

22. Even before this particular study was begun at the WES, other test results had shown that tracks have a significant influence on certain vibration characteristics of a vehicle. A series of studies was completed by the Chrysler Corporation to determine the influence of tracks on the vibration in an M60A1 tank.[8-11] As a part of this program, a series of tests was conducted at the Chrysler Proving Grounds in Chelsea, Michigan, to provide some insight into the relative levels of vibration encountered in hard-surface operation and in rough cross-country terrain. Early in this series, the vibration levels in the rough cross-country terrain were found to be only about 10 percent of those in operations on the hard surfaces. Since the interest of these studies was primarily centered about the design and life cycle of structural components during normal operation, emphasis was focused on the high frequencies associated with component failures and optical equipment impairments. These frequencies are excited chiefly by the number of track pads striking the surface per second, and tend to create conditions that are adverse to firing and sighting, but not discomforting to the crew.

23. At about the same time that the Chrysler studies were being made, the WES and the U. S. Army Tank-Automotive Command began a joint development of a comprehensive computerized model for predicting the ground mobility of vehicles in a cross-country environment. One prominent feature in this model is a simulation technique for determining vehicle speed as limited by vibration. The interest here was not so much in the high frequencies associated with component malfunctions, but rather in the low-frequency undulations that are associated with hull motions during cross-country operation and that, under extreme conditions, limit the control and speed of the vehicle.

24. As a result of the foregoing circumstances, efforts were initiated to investigate the effect of tracks on the gross, low-frequency hull motions of the type encountered in rough-terrain operations, and to develop a model for tracked vehicle dynamics that would be as simple as possible and afford suitable simulations of cross-country vibrations.

Data describing the dynamics of an M60A1 tank crossing single obstacles that were 6, 8, 10, 12, 16, and 18 in. high were available from a recent study.[12] Therefore, the M60A1 tank was used as the model vehicle to obtain necessary parameters as well as some verification of prediction accuracy.

25. The tank is represented in the form of coupled, second-order differential equations that describe the motions of each degree of freedom. The equations derive naturally by applying Newton's second law to the mass-spring-damper elements representing the vehicle's components. The elements comprising the vibratory systems are idealized in the usual sense: the mass elements are assumed to be rigid bodies, the spring elements are assumed to be of a negligible mass and represent the elastic properties of the structure, and the damping elements are assumed to have neither mass nor elasticity and represent the dissipative forces or energy losses of the system. Damping forces exist only if there is relative motion between the two ends of the damper.

26. Although a vehicle is a very complicated vibrational system possessing many degrees of freedom, a good many of these are rather unimportant for many types of problems. As a result of a compromise involving model complexity, adequate description of the significant motions, and time and cost of computer simulations, a two-dimensional model was used to represent the tank.

27. A schematic of the system that was modeled is shown in fig. 2.

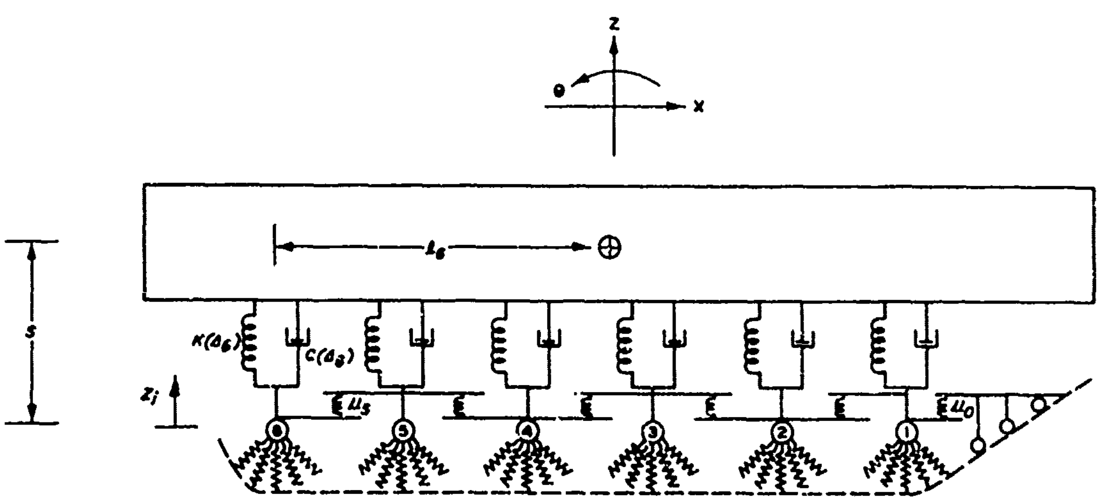


Fig. 2. Schematic of model of M60A1 tank

This model consists of eight degrees of freedom that include the bounce and pitch of the main frame and the vertical motions of each of the six bogie wheels. In addition, the motions in the vicinity of the driver are computed. The geometry effects of the bogies are represented by radially projecting stiff springs, and the track compliance is represented by interconnecting springs between the bogies and three "feelers," appropriately positioned in front of the first bogie to portray the geometry of the leading portion of the track and connected to it by a spring.

28. The longitudinal motion is accounted for only in the acceleration determined from the horizontal forces resulting from deflections of the bogie spring segments. No attempt is made to simulate the horizontal motions from a fixed reference. This method of accounting for horizontal accelerations is analogous to supplying the input force necessary for the actual tank to maintain a constant velocity while crossing an obstacle, and determining the acceleration from the additional force required to maintain this velocity.

Development of Equations

29. The differential equations describing the motions of each degree of freedom were developed by first establishing an appropriate set of coordinates and sign conventions, and then placing each system in a displaced configuration such that each coordinate was nonzero. The relative displacements of the masses cause compressions and extensions in the springs and relative motion of the damper ends that produce forces on each mass, as represented by the free-body diagram in fig. 3. Using Newton's second law of motion and summing first the vertical and longitudinal forces and moments on the main frame and then the vertical forces on each bogie led to the series of equations listed below to describe the M60A1 tank.

a. Forces and moments on main frame.

(1) Sum of vertical forces:

$$M\ddot{Z} = \sum_{i=1}^{6} k(\Delta_i)\Delta_i + \sum_{i=1}^{6} c(\dot{\Delta}_i)\dot{\Delta}_i - Mg$$

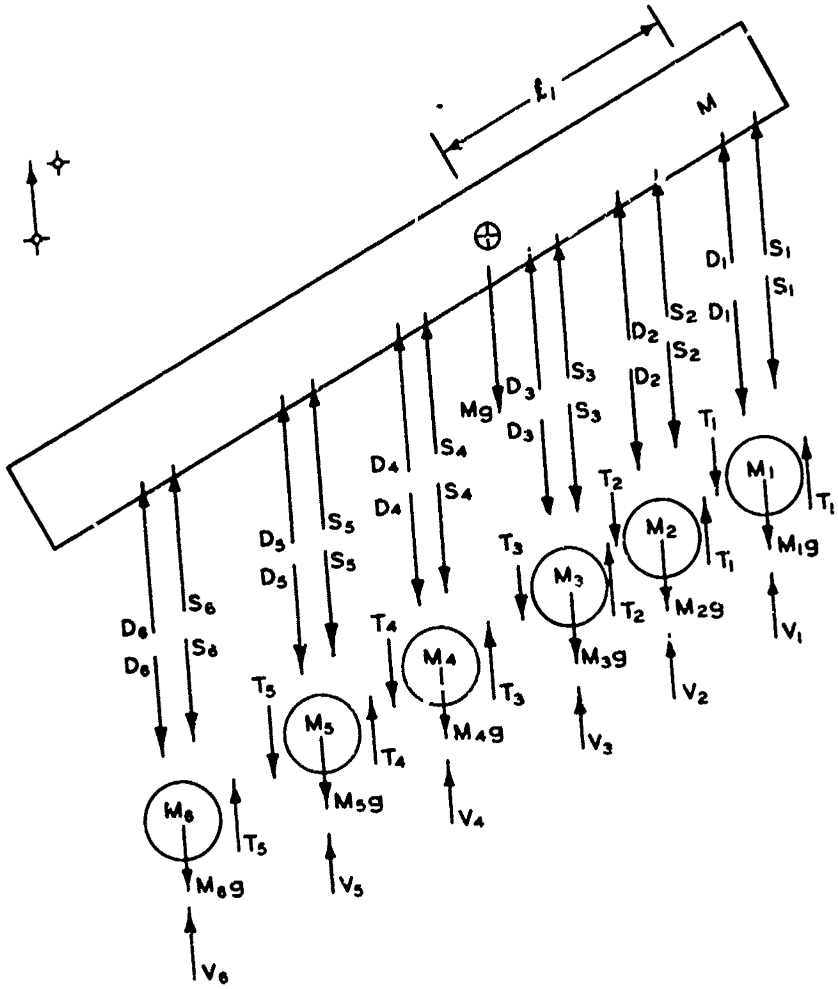


Fig. 3. Vertical forces acting on tank free body

(2) Sum of moments:

$$I\ddot{\theta} = -\left[\sum_{i=1}^{3} k(\Delta_i)\Delta_i \ell_i \cos\theta + \sum_{i=1}^{3} c(\dot{\Delta}_i)\dot{\Delta}_i \ell_i \cos\theta - \sum_{i=3}^{6} k(\Delta_i)\Delta_i \ell_i \cos\theta - \sum_{i=3}^{6} c(\dot{\Delta}_i)\dot{\Delta}_i \ell_i \cos\theta\right]$$

(3) Sum of horizontal forces:

$$M\ddot{x} = \sum_{i=1}^{6} H_i$$

b. Vertical forces on bogies.

$$M_1\ddot{z}_1 = k(\Delta_1)\Delta_1 + c(\dot{\Delta}_1)\dot{\Delta}_1 + \mu_1\delta_1 - \mu_0\delta_0 - M_1 g + V_1$$

$$M_2\ddot{z}_2 = k(\Delta_2)\Delta_2 + c(\dot{\Delta}_2)\dot{\Delta}_2 - \mu_1\delta_1 + \mu_2\delta_2 - M_2 g + V_2$$

$$M_3\ddot{z}_3 = k(\Delta_3)\Delta_3 + c(\dot{\Delta}_3)\dot{\Delta}_3 - \mu_2\delta_2 + \mu_3\delta_3 - M_3 g + V_3$$

$$M_4\ddot{z}_4 = k(\Delta_4)\Delta_4 + c(\dot{\Delta}_4)\dot{\Delta}_4 - \mu_3\delta_3 + \mu_4\delta_4 - M_4 g + V_4$$

$$M_5\ddot{z}_5 = k(\Delta_5)\Delta_5 + c(\dot{\Delta}_5)\dot{\Delta}_5 - \mu_4\delta_4 + \mu_5\delta_5 - M_5 g + V_5$$

$$M_6\ddot{z}_6 = k(\Delta_6)\Delta_6 + c(\dot{\Delta}_6)\dot{\Delta}_6 - \mu_5\delta_5 - M_6 g + V_6$$

where (for all the above equations)

M, M_i = mass of main frame and i^{th} bogie assembly, respectively

$\ddot{z}, \dot{z}, z$ = vertical motions at center of gravity of main frame, i.e. acceleration, velocity, and displacement, respectively

$\ddot{z}_i, \dot{z}_i, z_i$ = vertical motions at center of gravity of the i^{th} bogie, i.e. acceleration, velocity, and displacement, respectively

$\ddot{\theta}, \dot{\theta}, \theta$ = angular motion at the center of gravity of the main frame

$\ddot{x}$ = horizontal acceleration at center of gravity of main frame

$\Delta_i = z + \ell_i \sin\theta - z_i$ for $1 \leq i \leq 3$

$= z - \ell_i \sin\theta - z_i$ for $4 \leq i \leq 6$

$\dot{\Delta}_i = \dot{z} + \ell_i\dot{\theta} \cos\theta - \dot{z}_i$ for $1 \leq i \leq 3$

$= \dot{z} - \ell_i\dot{\theta} \cos\theta - \dot{z}_i$ for $4 \leq i \leq 6$

ℓ_i = distance from center of gravity of main frame to contact point of the i^{th} bogie

$k(\Delta_i)$ = force-deflection relation for i^{th} bogie suspension

$c(\dot{\Delta}_i)$ = force-velocity relation for i^{th} bogie suspension

g = acceleration of gravity

I = pitch moment of inertia of main frame

H_i = resultant horizontal force of spring segments of i^{th} bogie

μ_i = spring constant for i^{th} track spring; in this study, μ_0 = 600 lb/in., and μ_i = 375 lb/in. for $1 \leq i \leq 5$

$\delta_i = z_{i+1} - z_i$ = relative displacement between adjacent bogies

V_i = resultant vertical force of spring segments of i^{th} bogie

30. Representative force-deflection and force-velocity relations for describing suspension compliance are shown in figs. 4a and 4b, respectively. The rotational displacements and velocities of the road arms have been converted to translational motions, and the relations include all of the suspension's nonlinearities, including the jounce and rebound limits. Once the motions at the center of gravity of the main frame have been determined, the motions in the vicinity of the driver can be determined in the usual manner by combining the translational and rotational motions.

Track forces

31. The track compliance is represented chiefly by interconnecting linear springs between the bogies and massless feelers that are connected to the front bogie by a stiff spring. The spring constants portraying the track tension were determined by observing photographs of the M60A1 tank in different positions on an obstacle (fig. 5). From these photographs, the influence of displacing a particular bogie on the displacement of the adjacent bogies was estimated. With the approximate mass of each bogie assembly and their displacements relative to each other and the main frame known, an appropriate spring constant could be determined as follows (refer to fig. 6):

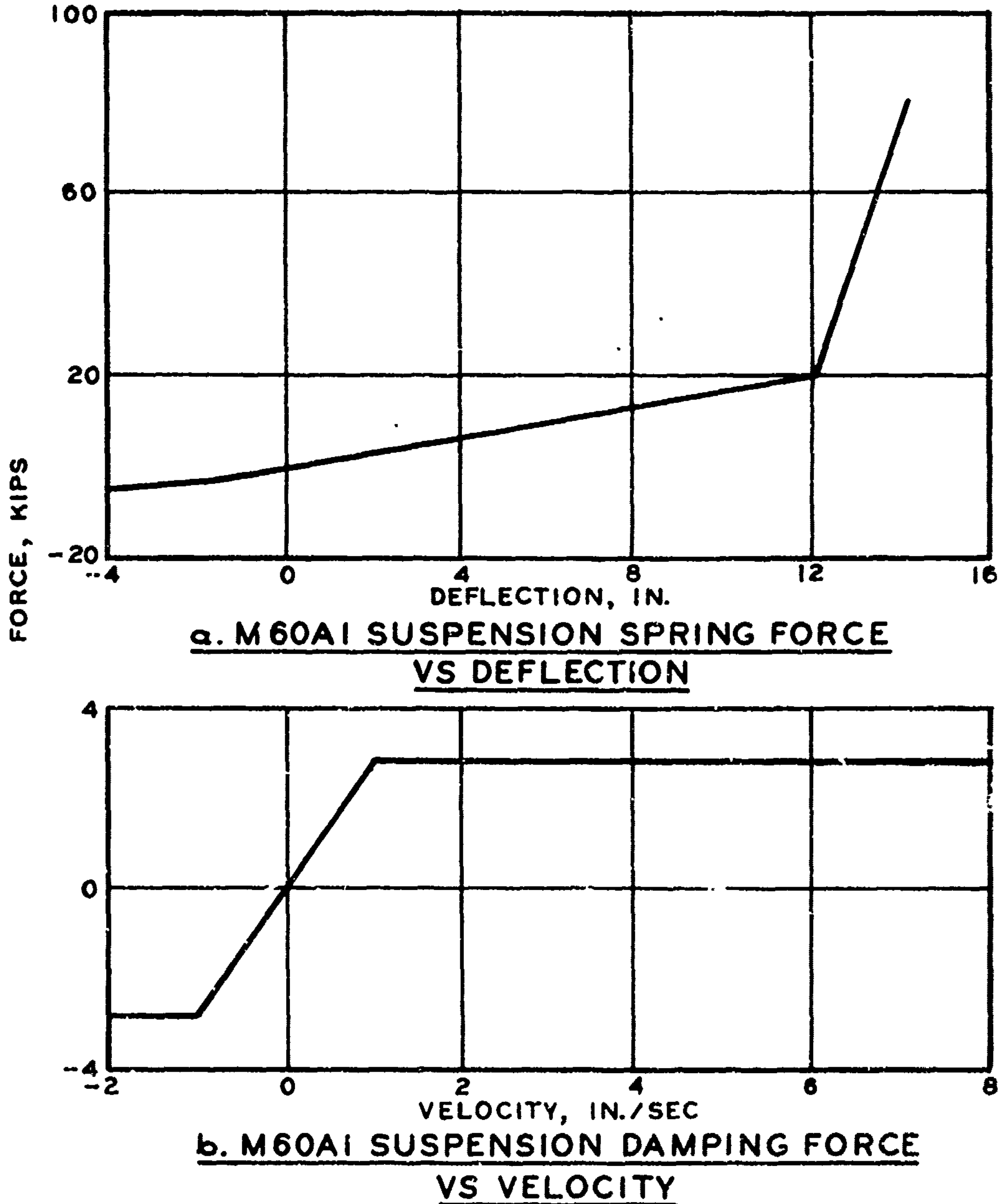
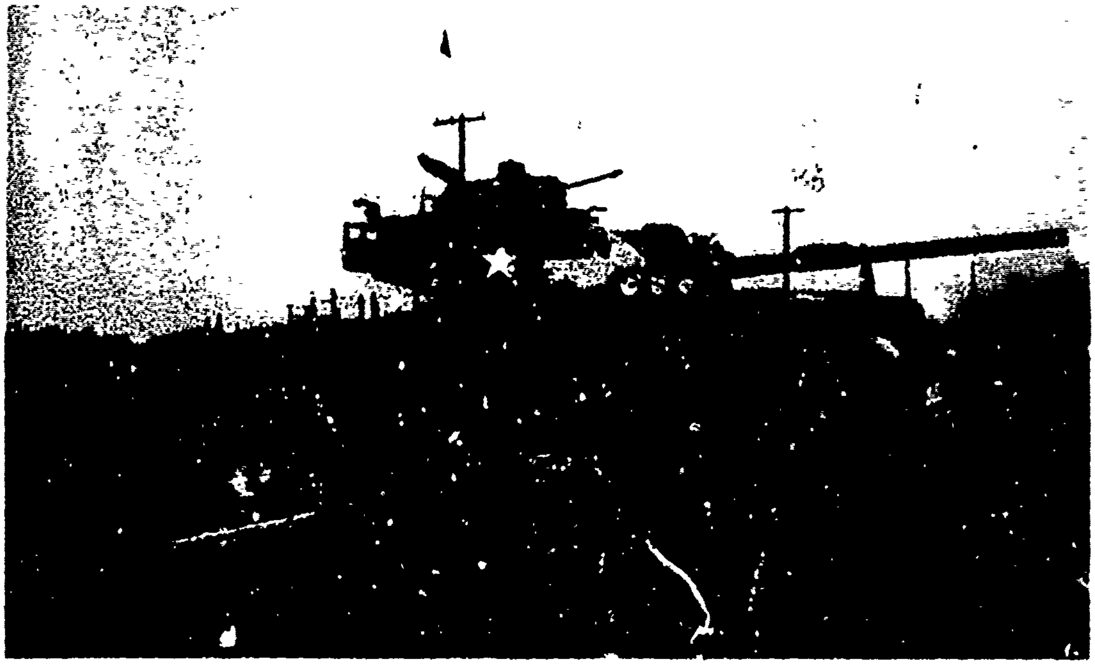
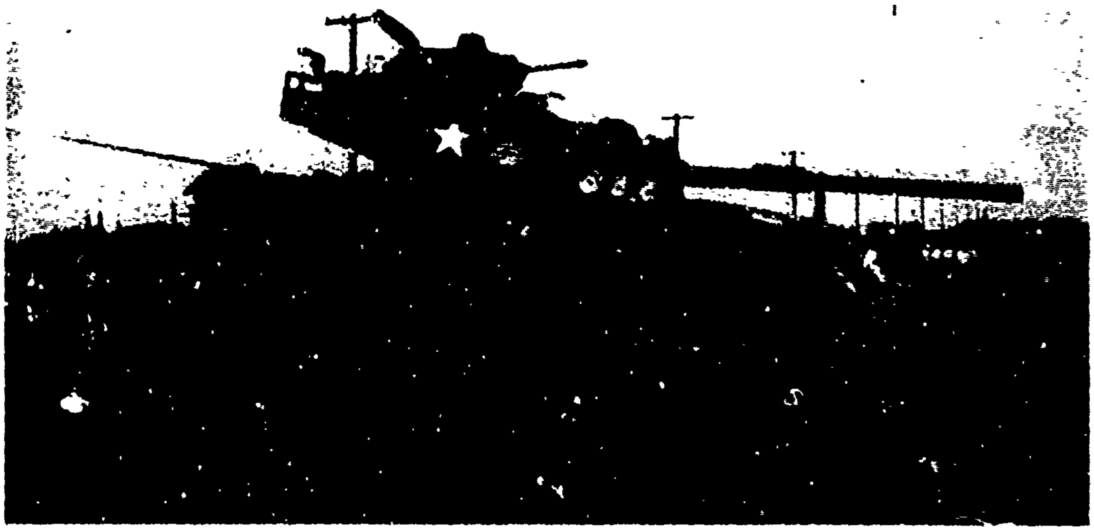


Fig. 4. Suspension compliance of M60A1 tank



a. 12-in.-high obstacle; 2 mph



b. 12-in.-high obstacle; 10 mph

Fig. 5. Relative displacements of bogies as M60A1 crosses obstacle

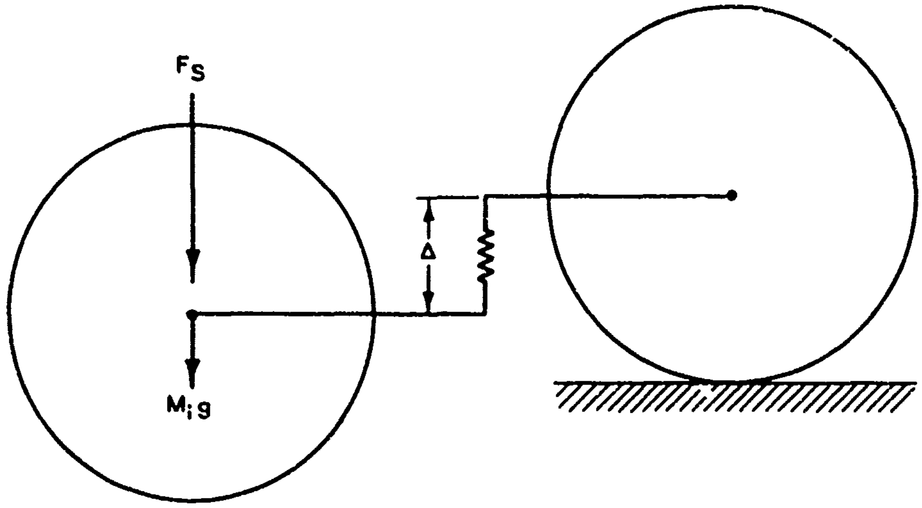


Fig. 6. Schematic for use in determining track-spring constant

General equation: $F = K\Delta$

where

F = applied force

Δ = spring deflection

K = spring constant

$$F = F_s + M_i g$$

where

F = total applied force on bogie

F_s = resultant force due to suspension reaction

$M_i g$ = force due to weight of i^{th} bogie

$$\text{Track-spring constant} \quad K = \frac{F}{\Delta} = \frac{F_s + M_i g}{\Delta}$$

where Δ is relative displacement between adjacent bogies

32. Close observation further revealed that when the M60A1 approached an obstacle larger than about 6 in. high, the initial track-obstacle contact tended to lift the front bogie and guide it over the obstacle. This lifting had a significant effect on the longitudinal

motion. To simulate this effect, massless feelers were positioned in front of the first bogie, each at a different threshold height, to conform to the geometry of the leading portion of the track. The influence of the feelers in lifting the front bogie depends on the height and shape of the encountered obstacle. Again, from observing photographs of initial contacts with obstacles and analyzing the relative track-bogie displacements, an effective spring constant of 600 lb/in. was obtained. (Of course, the proper way to determine accurate spring constants is by the use of potentiometers interconnected between bogies and connected to the main frame.)

Bogie spring segment constants

33. The segmented wheel concept[13] was used in the track model to (a) provide the flexibility for predicting longitudinal accelerations (if needed), (b) include the important effects of the bogie geometry, and (c) incorporate a means for accounting for the effects of the envelopment characteristics of the tracks. Each bogie was divided into twelve 10-deg segments, six on each side of the vertical position, as shown in fig. 7. To account for track thickness, 2 in. was added to the length of each radius of each bogie.

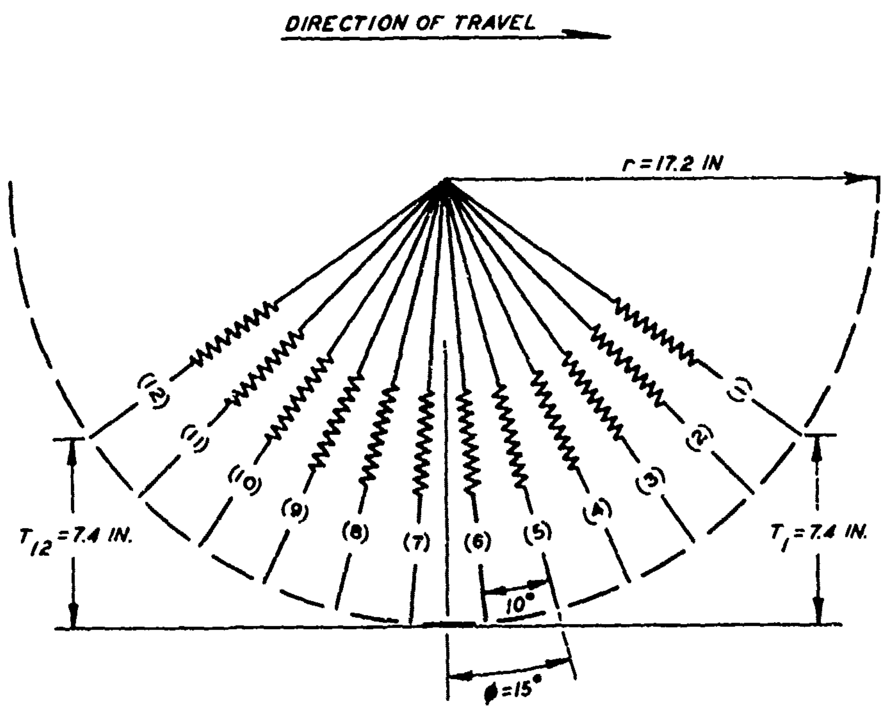


Fig. 7. Schematic of bogie spring segment configuration

34. To compute the deflections of each spring segment, the segment threshold heights T_i must first be computed. These heights are simply the heights from the ground to each spring of the undeflected bogie (fig. 7). The segment deflection δ_i^* is then computed by the equation

$$\delta_i^* = \begin{Bmatrix} Y_i - T_i - z_i \ , & Y_i - T_i - z_i \geq 0 \\ 0 \ , & Y_i - T_i - z_i < 0 \end{Bmatrix}$$

where

Y_i = vertical obstacle height beneath the i^{th} segment

z_i = vertical axle displacement of i^{th} bogie

35. The segment deflections are permitted to have positive values only; negative values are replaced by zero. The reference from which vertical displacements are measured is the point that locates the axle when the bogie is imagined to be rigid and in static equilibrium. Static deviations from this reference correspond to static bogie deflections, and superposed on these static deflections are the dynamic obstacle-induced deflections.

36. The spring constants for the bogie segments were obtained in the following manner. The periphery of the bogies was encased in a hard, abrasive-resistant rubber shell approximately 2 in. thick. A channel down the center divided this shell into two bands, each approximately 5 in. wide. The first step in determining segment constants was to consider the relations between linear force-deflection and stress-strain curves of the type shown in fig. 8. The equation describing force F and deflection δ is

$$F = K\delta \tag{1}$$

The equation describing stress σ and strain ε is

$$\sigma = E\varepsilon \tag{2}$$

The idea is to obtain a relation between the constants of proportionality

K (spring constant) and E (modulus of elasticity). By defining $\sigma = F/A$ and $\epsilon = \Delta/L - \delta/L$, equation 2 can be written

$$F = EA\frac{\delta}{L} \tag{3}$$

Equating the forces in equations 1 and 3 yields the desired relation between K and E :

$$K = \frac{EA}{L} \tag{4}$$

where

K = spring constant, lb/in.

E = modulus of ela.ticity, psi

A = the area upon which pressure is being applied, sq in.

L = thickness of rubber casing, in.

37. The thickness L was taken as 2 in. for each bogie. The effective area was determined to be that portion of the rubber shell beneath the bogie hub upon which pressure was being applied; it was computed to be approximately 20 sq in. A value of 500 psi, obtained from a handbook of material properties for a hard, abrasive-resistant rubber, was used for the modulus of elasticity.

38. Substituting these values into equation 4 yielded a spring constant of 5000 lb/in. A schematic of the bogie shown in fig. 9 illustrates the manner in which these values were obtained.

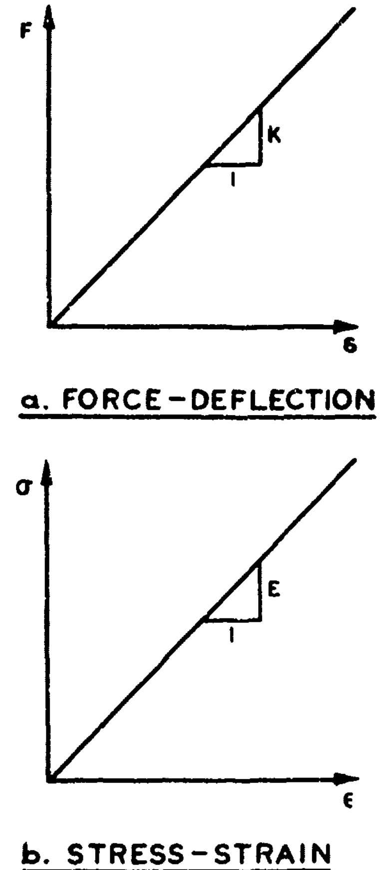


Fig. 8. Force-deflection and stress-strain relations

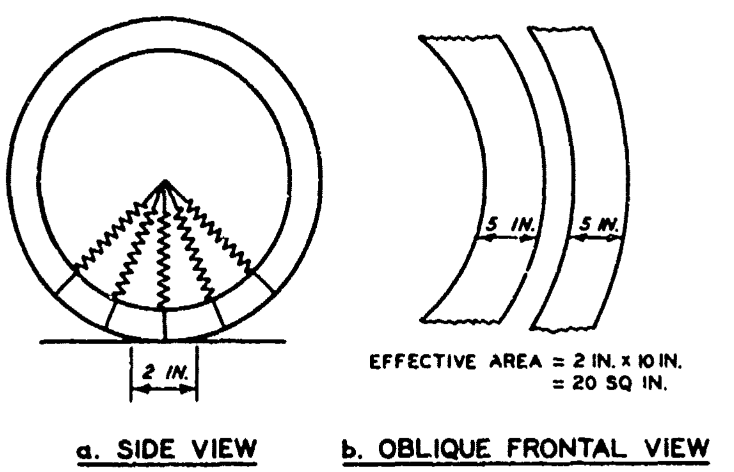


Fig. 9. Schematic illustrating effective areas of pressure application of M60A1 bogie

Vertical and horizontal
forces on first bogie axle

39. The resultant vertical and horizontal forces on the first bogie axle due to the spring segment deflections are given by the equations

$$V_1 = \sum_{i=1}^{12} \left(K \cos \phi_i\right)\delta_i^*$$

$$H_1 = \sum_{i=1}^{12} \left(K \sin \phi_i\right)\delta_i^*$$

where

K = the segment spring constant

ϕ_i = the angle of the i^{th} segment from the vertical

δ_i^* = the vertical deflection of the i^{th} segment

Method of Analysis and Evaluation of Predictions

40. Predictions of peak vertical and peak longitudinal accelerations were obtained from simulation for 20 obstacle height-impact speed combinations to compare with data from actual tests. In this study, vehicle performance was described only in terms of peak vertical acceleration-speed relations, and speed-obstacle height relations for the 2.5-g selected tolerance level. Peak vertical acceleration-speed relations for several obstacle heights were developed from measured and predicted data to provide a comparison for verification. These relations express the peak vertical acceleration to be expected when an M60A1 tank traverses an obstacle of a specific size at a given speed; and from them, the speed-obstacle height relations were developed to express the speed at which the tank can traverse a given obstacle without exceeding the 2.5-g tolerance level.

41. The prediction accuracy of the model was evaluated by comparing (a) the predicted and measured peak vertical acceleration-speed relations (fig. 10), and (b) speeds at 2.5-g peak vertical acceleration levels developed from predicted relations and from the measured data (fig. 11).

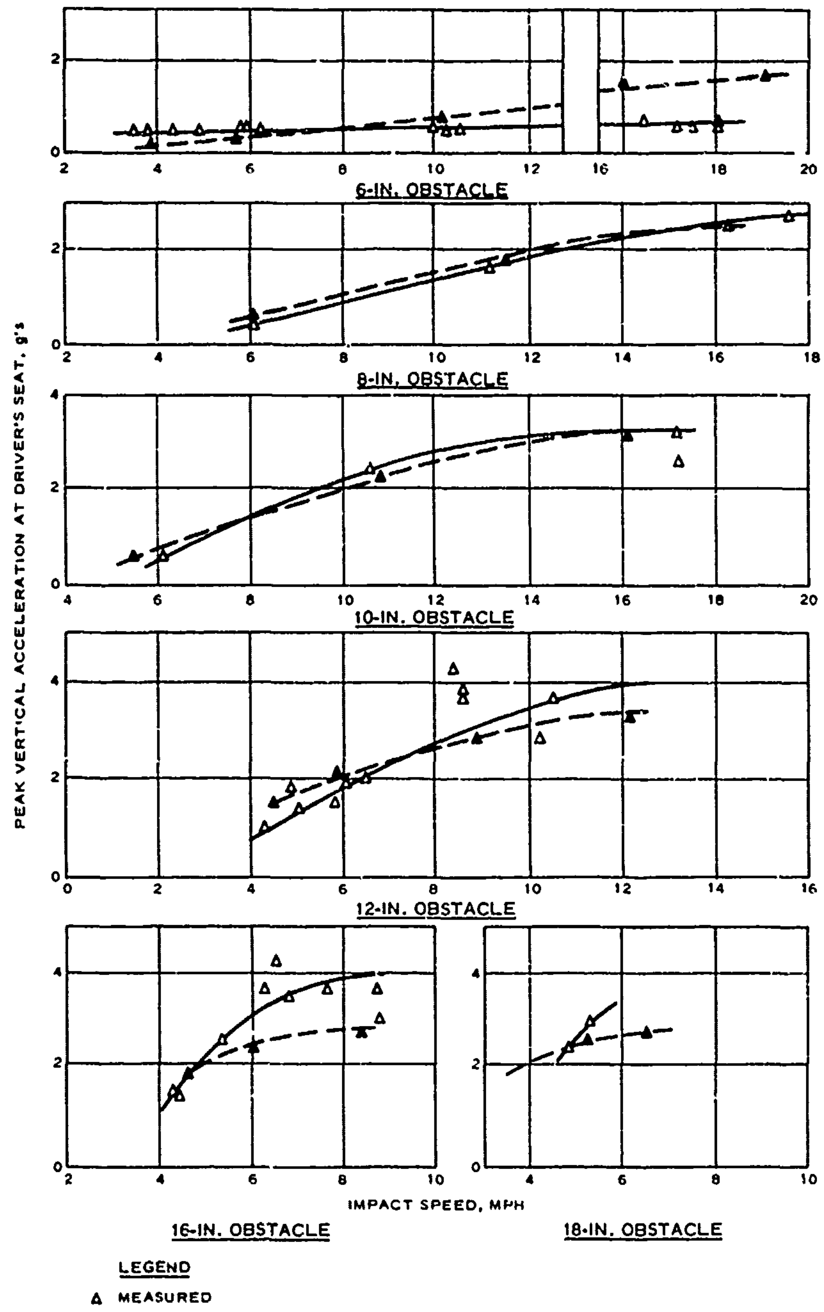


Fig. 10. Comparison of measured and predicted vertical accelerations at the driver's seat, M60A1 tank

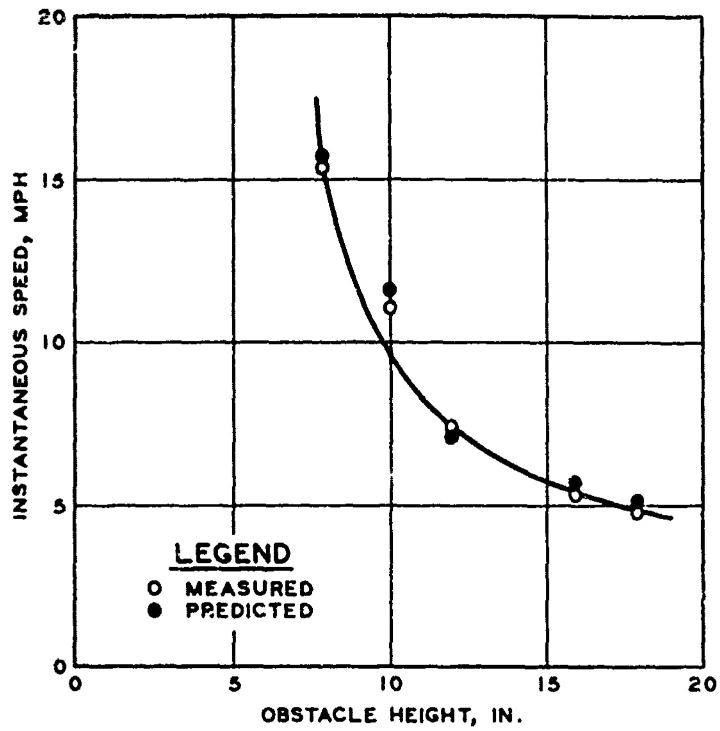


Fig. 11. Speed-obstacle height relations at 2.5-g peak vertical acceleration

42. The curves in fig. 10 represent the lines of best visual fit. Both measured and predicted peak vertical accelerations at the driver's seat increased with an increase in speed. There appears to be a tendency for the curves to crest at the higher speeds, suggesting that, after a critical speed has been reached, a further increase in speed would not result in an increase in peak vertical acceleration. Except for the 6-in.-high obstacle, the peak vertical acceleration that had been predicted was higher than that measured at low speeds, and lower than that measured at the higher speeds. The reverse is indicated for the 6-in.-high obstacle. The agreement of the measured and predicted curves for the 8- and 10-in. obstacles is very good; and at the 2.5-g tolerance limit, the curves for the 12-, 16-, and 18-in. obstacles are reasonably close.

43. The speed-obstacle height relation in fig. 11 was established from values of speed and corresponding values of obstacle height at which 2.5-g peak vertical acceleration occurred. The speed at which this acceleration is reached at the driver's seat decreases with an increase in obstacle height, and the effect of obstacle height on 2.5-g peak vertical acceleration begins to diminish rapidly at about the 9-in. obstacle height. The predicted data points are in good agreement with curves developed from the measured data. No field data were available to compare vibration in cross-country runs; this will be the next step in verifying and refining the model.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

44. Based on this study, the following conclusions were drawn:

   a. There are no ranges of vehicle characteristics, velocities, or obstacle sizes sufficiently broad to permit the neglect of track contributions to hull dynamics (paragraph 21).

   b. Within the range of speeds studied, contributions of the track were usually most pronounced at low speeds and of lesser importance at the higher speeds (paragraph 20).

   c. The nature of the suspension system significantly affects the track contribution to dynamics (paragraph 19).

   d. The mathematical model for tracked vehicle dynamics shows promise as a practical means for simulating pertinent hull dynamics of tracked vehicles traveling off roads.

Recommendations

45. It is recommended that:

   a. The mathematical model be verified by using cross-country field data.

   b. Methods for parameter determination in the mathematical model be refined.

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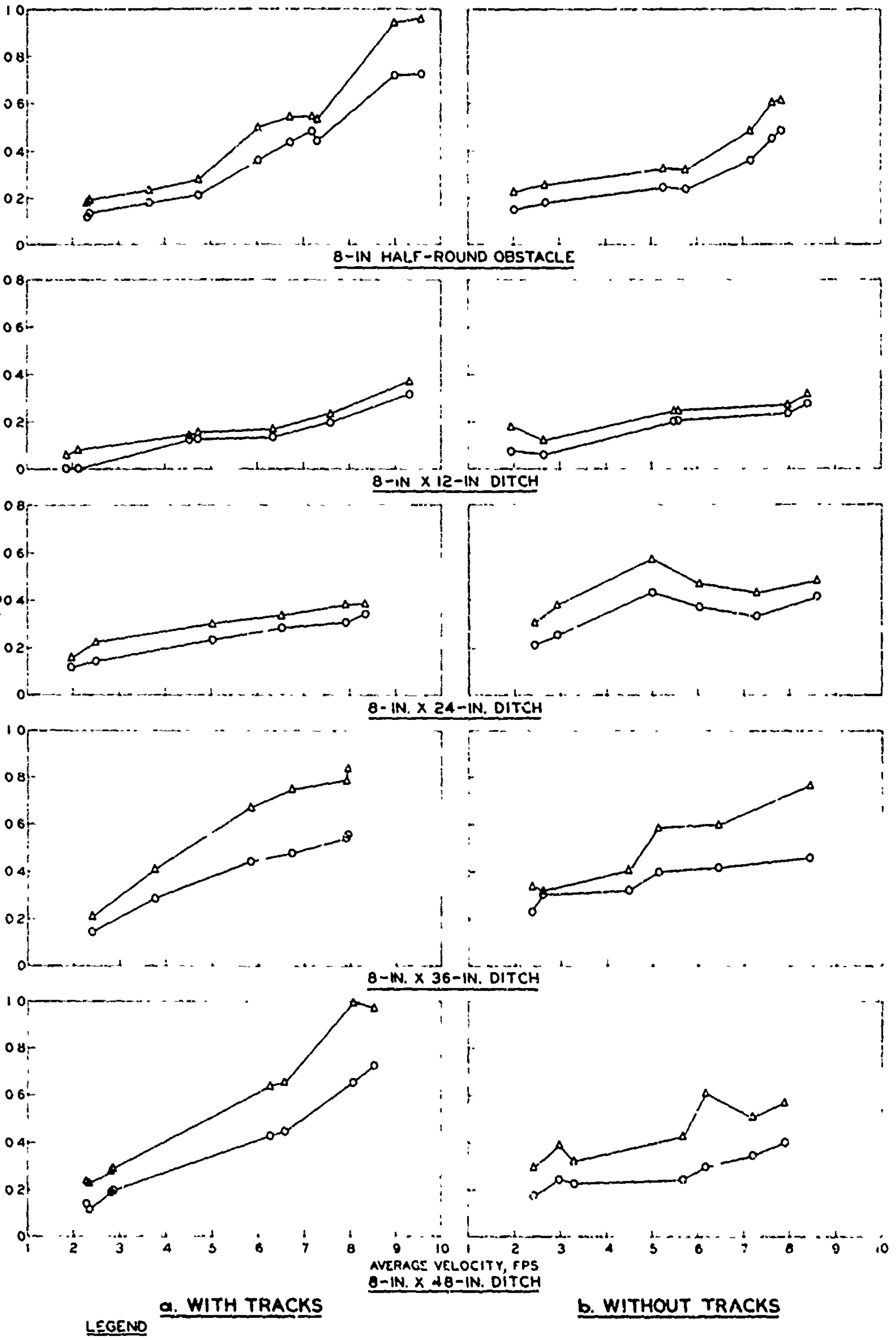
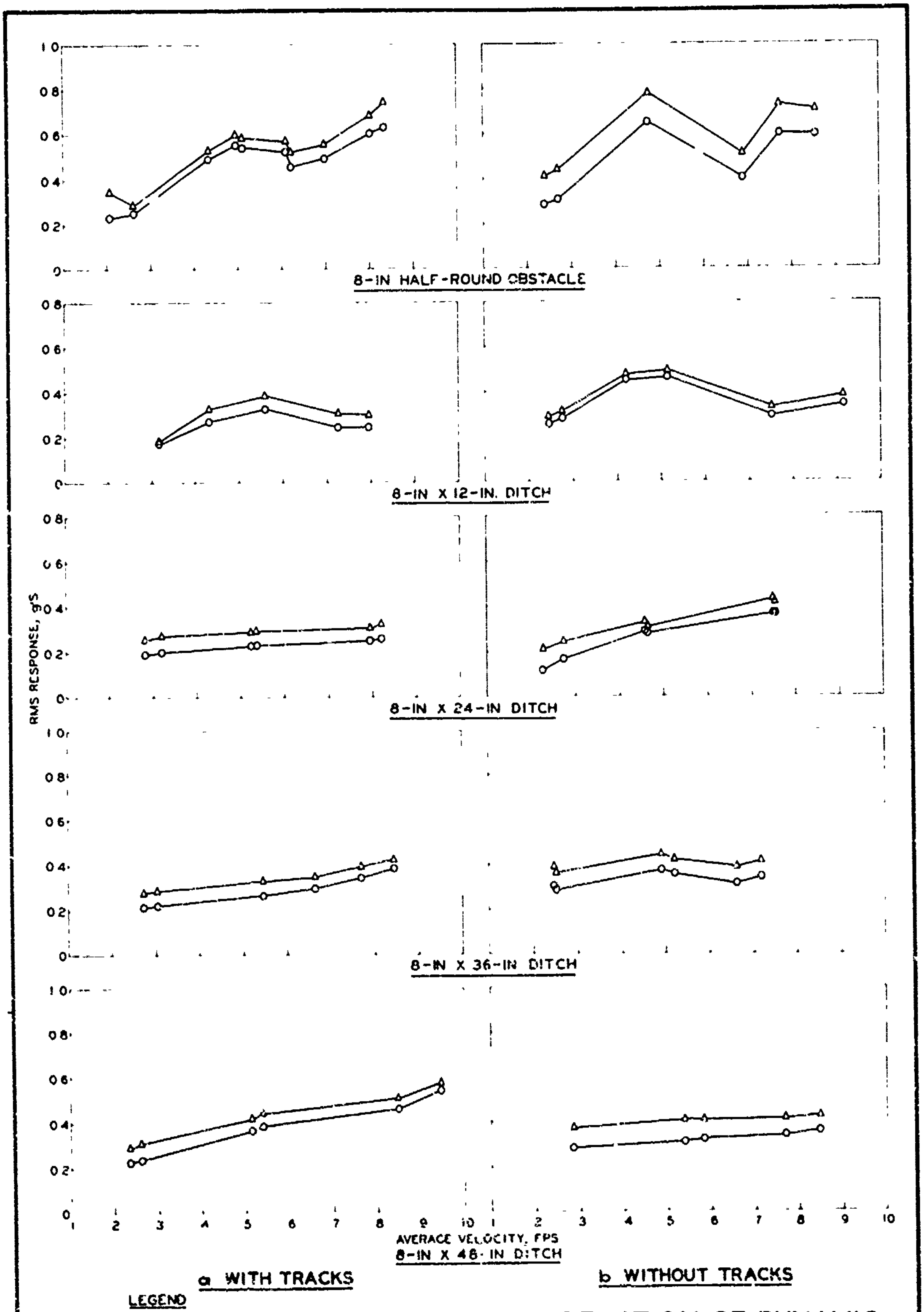


RELATION OF DYNAMIC RESPONSES TO AVERAGE VELOCITY FOR M29



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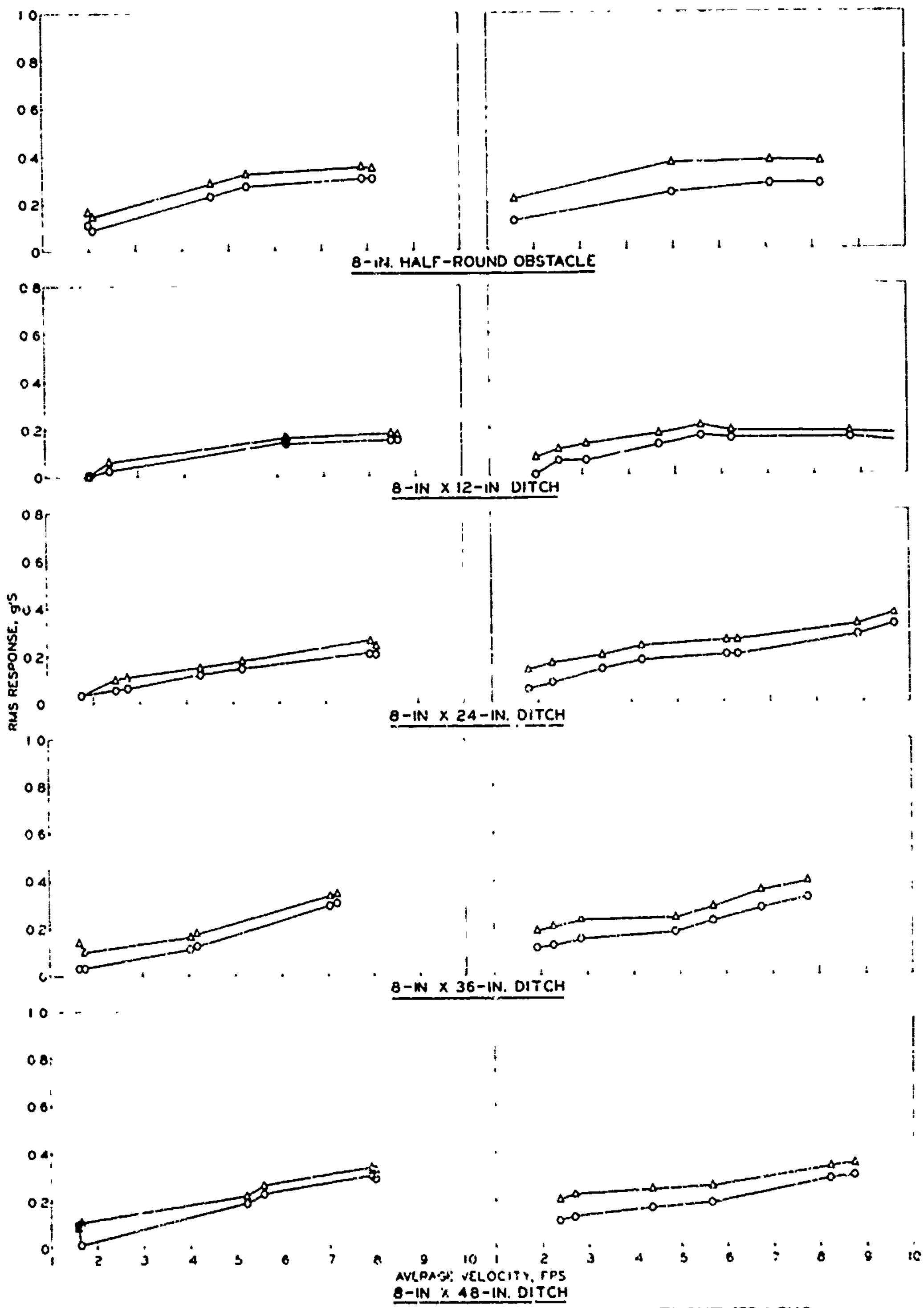
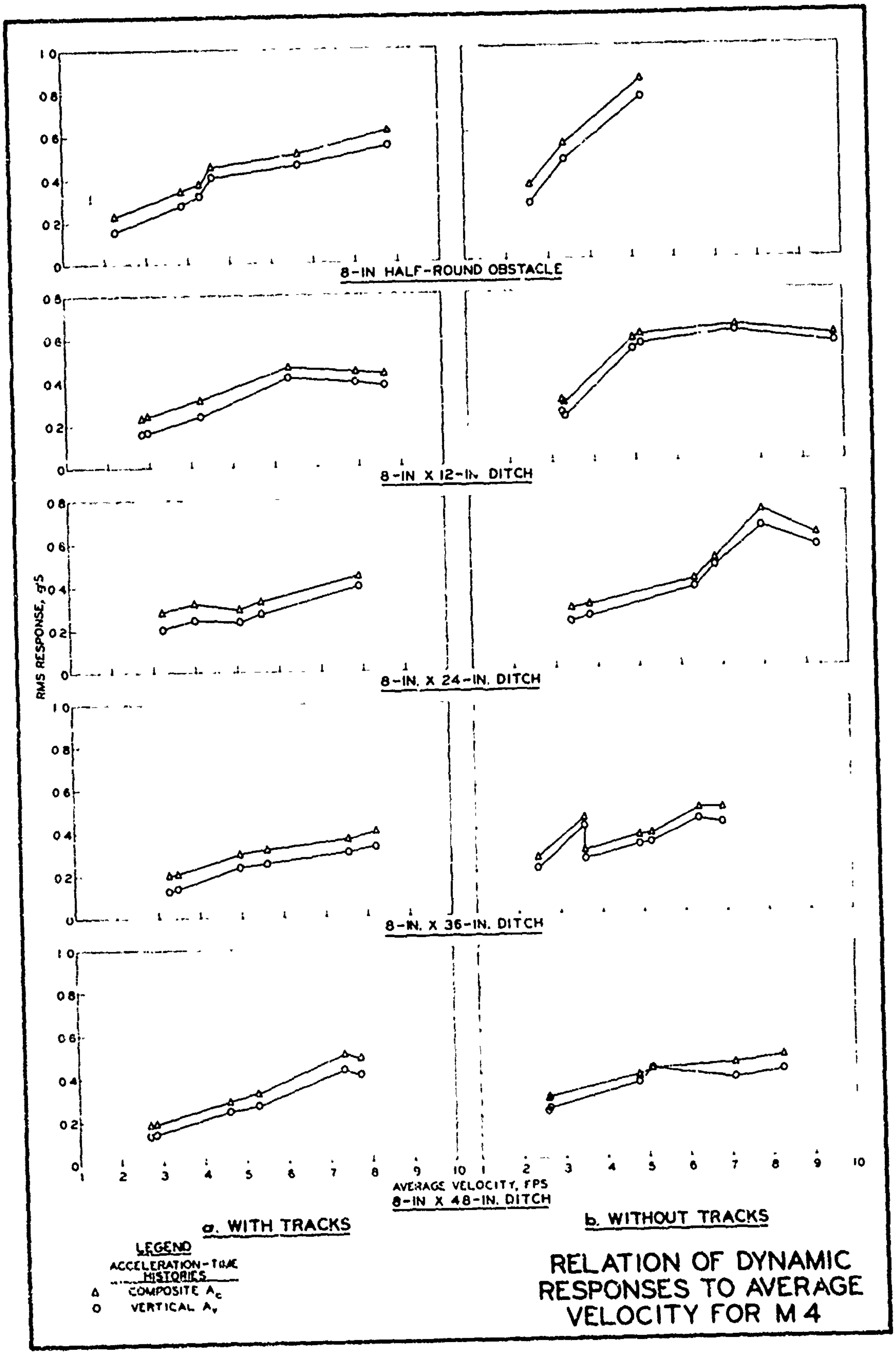
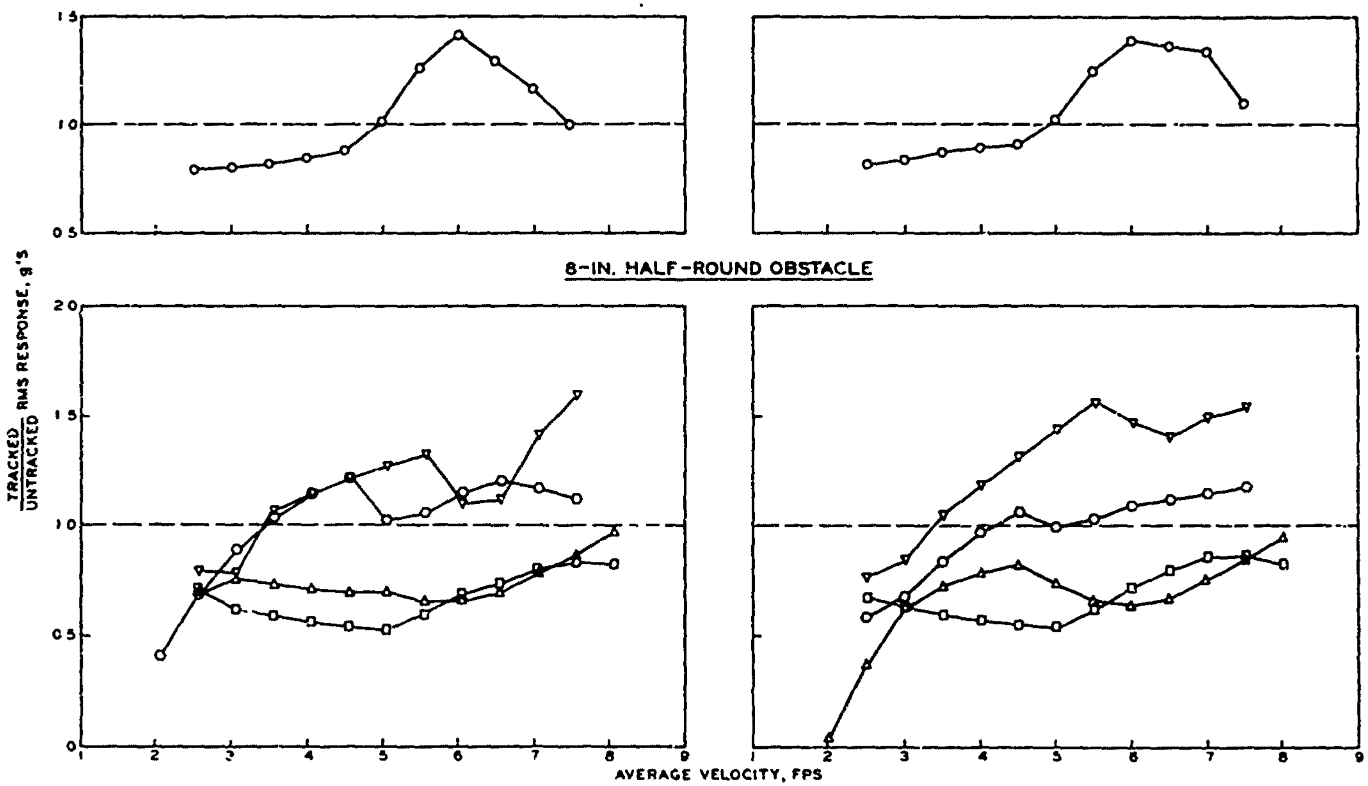


RELATION OF DYNAMIC RESPONSES TO AVERAGE VELOCITY FOR M113



RELATION OF DYNAMIC RESPONSES TO AVERAGE VELOCITY FOR M 4

PLATE 4



DITCHES

a COMPOSITE ACCELERATION-TIME HISTORIES

b. VERTICAL ACCELERATION-TIME HISTORIES

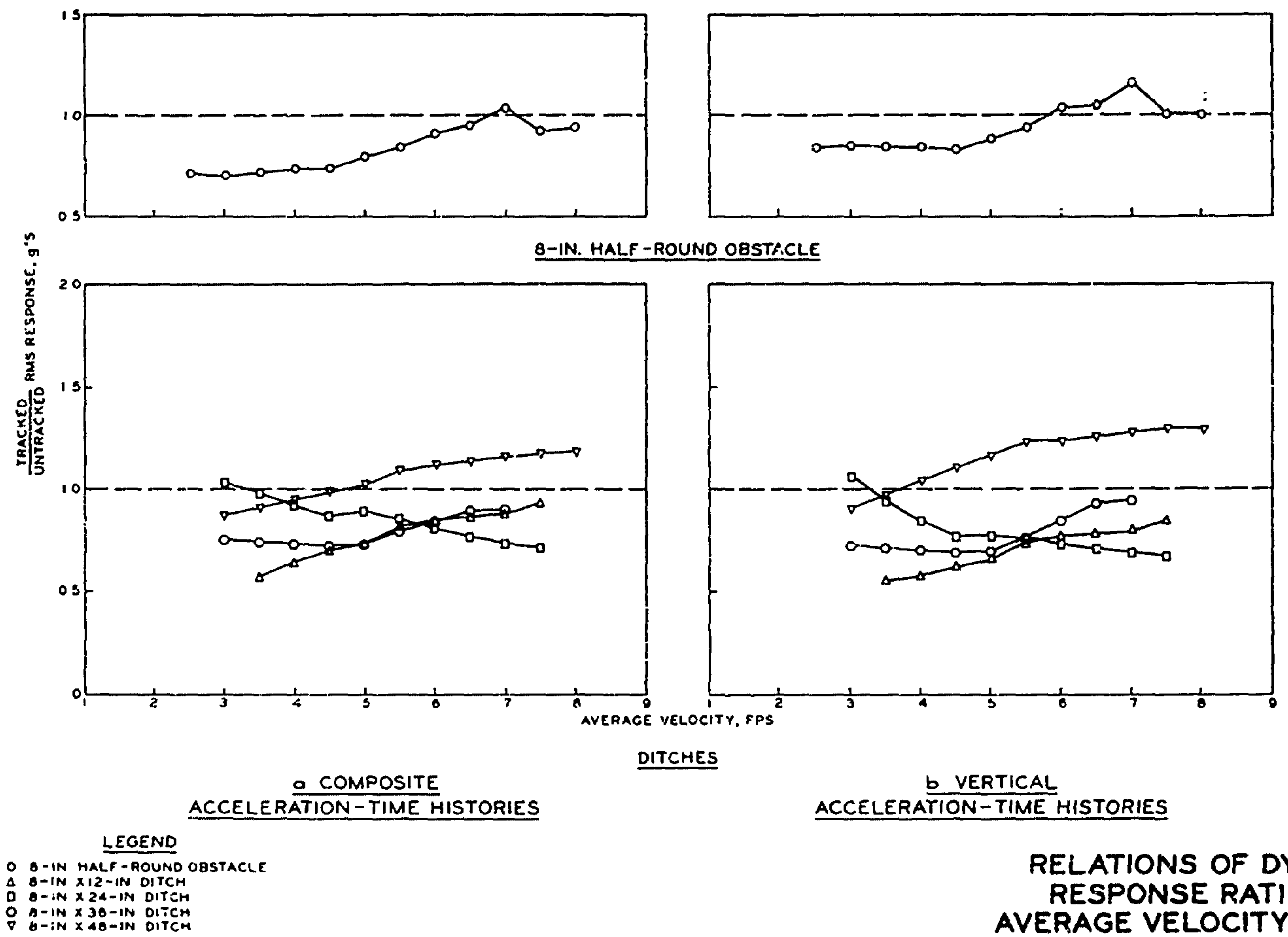
LEGEND

- O 8-IN HALF-ROUND OBSTACLE
- Δ 8-IN X 12-IN DITCH
- □ 8-IN X 24-IN DITCH
- ⬡ 8-IN X 36-IN DITCH
- ▽ 8-IN X 48-IN DITCH

RELATIONS OF DYNAMIC RESPONSE RATIOS TO AVERAGE VELOCITY FOR M29

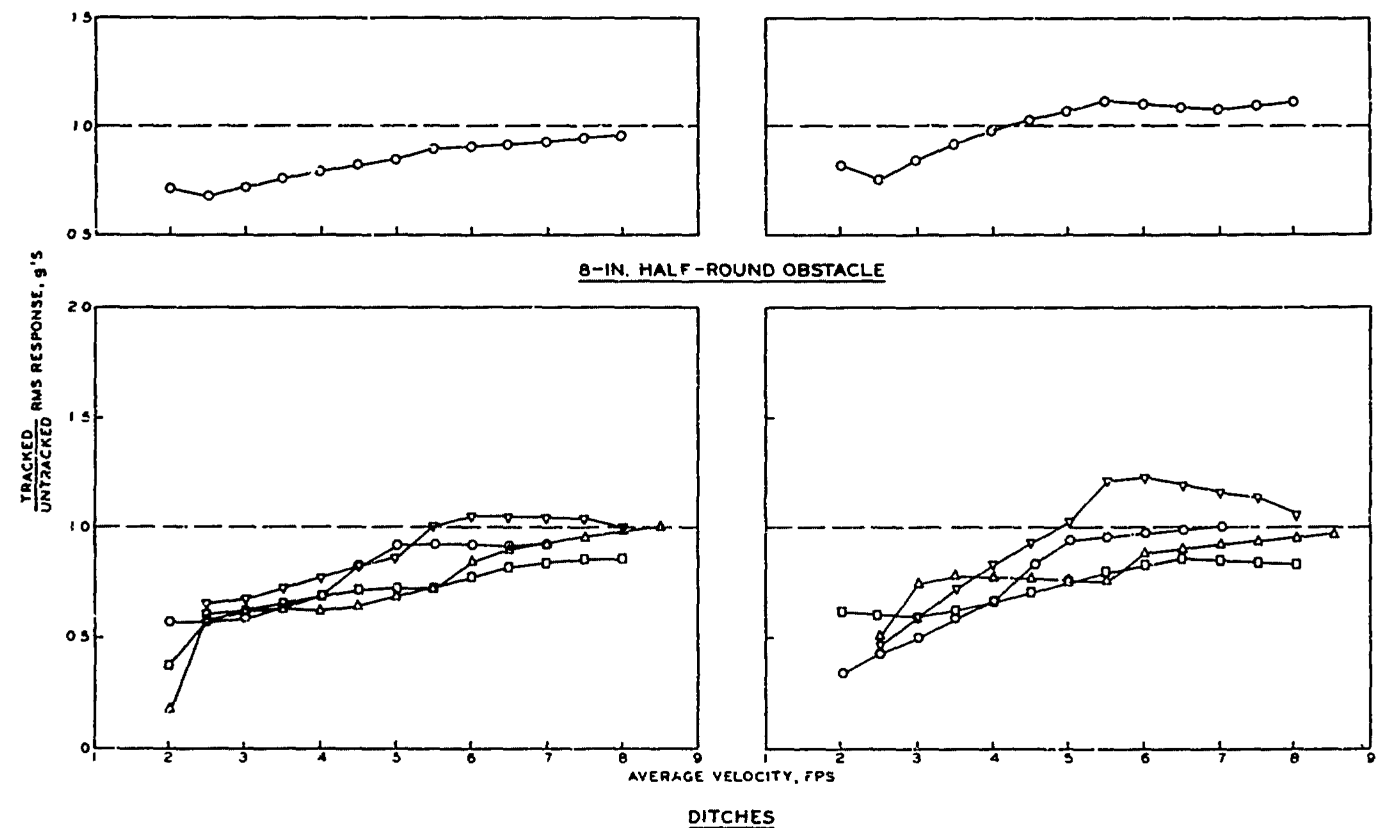
36

PLATE 5



RELATIONS OF DYNAMIC RESPONSE RATIOS TO AVERAGE VELOCITY FOR M114

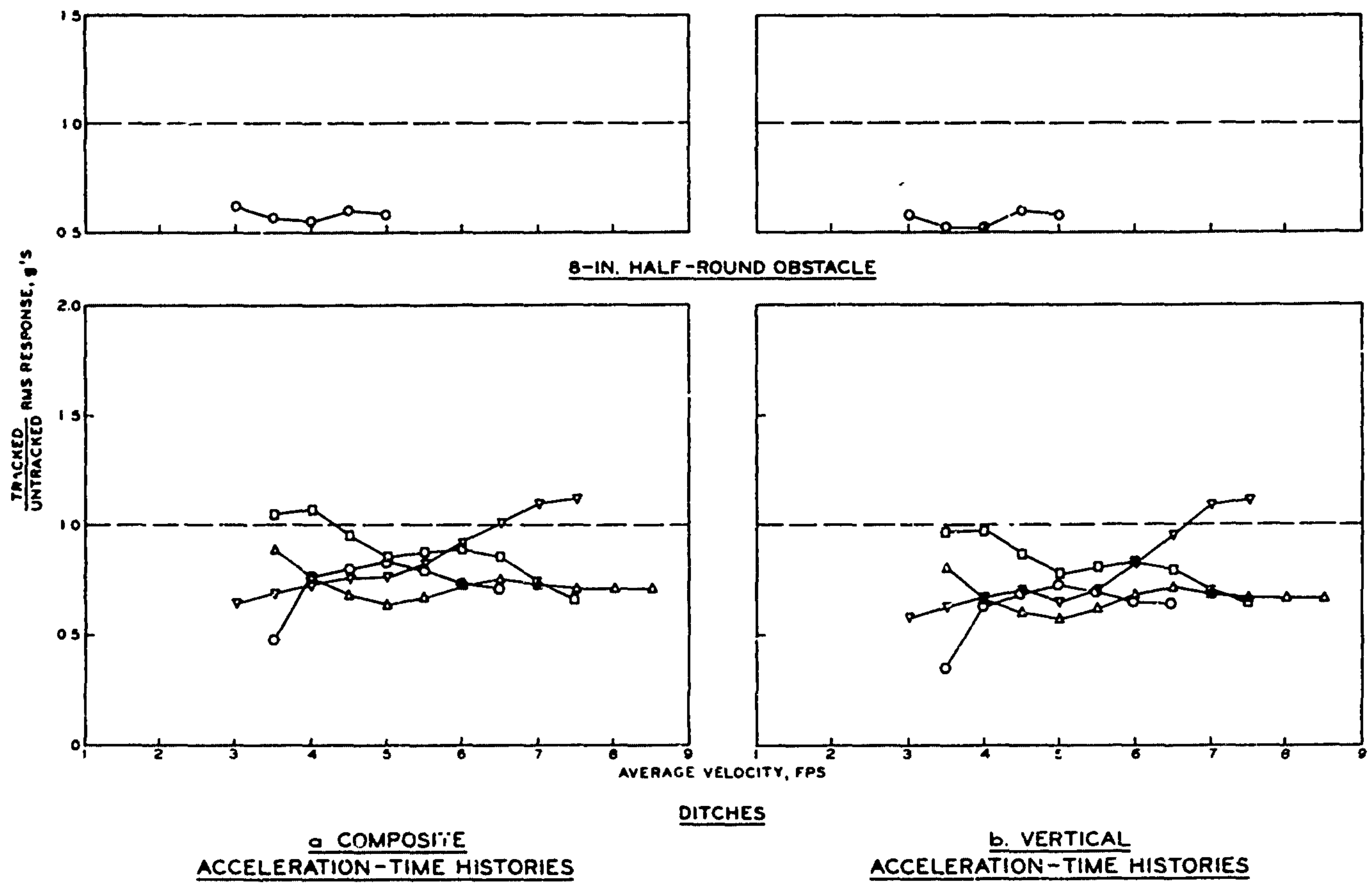
PLATE 6



a. COMPOSITE ACCELERATION-TIME HISTORIES

b. VERTICAL ACCELERATION-TIME HISTORIES

RELATIONS OF DYNAMIC RESPONSE RATIOS TO AVERAGE VELOCITY FOR M113



LEGEND

- O 8-IN HALF-ROUND OBSTACLE
- Δ 8-IN. X 12-IN DITCH
- □ 8-IN X 24-IN DITCH
- O 8-IN. X 36-IN DITCH
- ▼ 8-IN X 48-IN DITCH

RELATIONS OF DYNAMIC RESPONSE RATIOS TO AVERAGE VELOCITY FOR M4

PLATE 8